Science Supplementary Material
S383 The Relativistic Universe

S383

Errata 2013 onwards

This document lists errors identified during presentations of the module from 2013 onwards. These generally apply whether you have original versions of the books (1.1) or the reprinted versions (1.2). (The version number can be found on the page opposite the first contents page of each book. Note that both versions have the same published/copyright year 2010.) New errors identified during the current presentation of this module will be added directly to the Errata section of the website and not to this document.

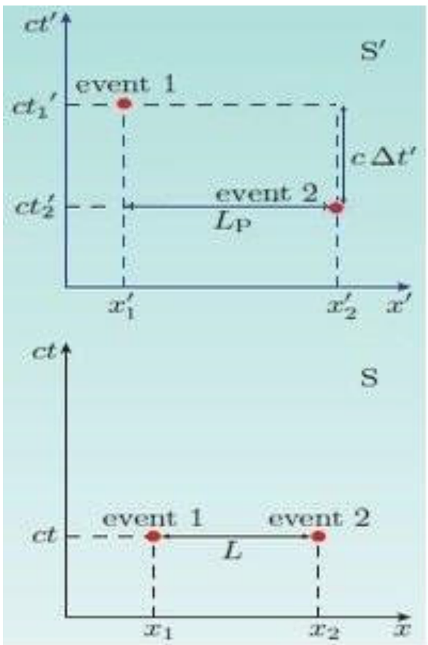
Relativity, Gravitation and Cosmology

On the cover page that identifies the author and consultant authors and in the last paragraph on page 9, Aiden Droogan should read Aidan Droogan.

P.19: In the caption to Figure 1.5 the last sentence should read: 'Note that $\gamma \geq 1 \ldots$'.

P.19: The sentence following Equation 1.11 should read: 'where we use the symbol $[x^\nu] \ldots$' (i.e. the superscript should be ν and not μ). The same replacement should be made in the paragraph immediately below Equation 1.12 (twice) and in the first line of the paragraph below that.

P.26: There is an error in the top panel of Figure 1.9. The following sentence should also be added to the end of the figure caption: '(As you will see later, $\Delta t'$ is negative in this case, so t'_2 has been shown to be less than t'_1.)' A revised figure is shown below:



P.65: In line 1, the covariant four-displacement should read: $(c\Delta t, -\Delta \mathbf{r})$.

P.89: In line 8 of the second paragraph (below Figure 3.10), the expression for the radius of the cylinder should read: $R = (b - a)/\,2\pi$.

P.105: In the paragraph above Equation 3.36, it should read: 'However, because of the definition of the connection coefficients . . .'.

P.107: In Exercise 3.18, delete the reference to 'Minkowski', so that it reads 'A two-dimensional spacetime ...'.

P.108: In the ninth summary point, second line of text, remove the summation symbol $\sum_{i,j}$ from in front of $[g^{ij}][g_{ij}]$.

P.124: In the second paragraph from the bottom, delete 'so $\boldsymbol{e}_r \cdot \widehat{\boldsymbol{n}} = -1$' so that the sentence now ends with '. . . because in this case the field points inwards.'

P.130: In the text below Equation 4.29, the spatial derivatives of the energy flows should be given as T^{j0} and not T^{0i}.

P.137: The definition of the norm in the 4th line down should read $\Sigma_{\alpha,\beta} g_{\alpha\beta} t^\alpha t^\beta$ – i.e. there shouldn't be a comma in the metric $g_{\alpha\beta}$.

P.155: In Section 5.3.1, the second sentence should read '. . . inertial frames are . . .'.

P.163: In the paragraph below Equation 5.27 it should read 'In particular, we noted earlier that the stationary nature ...' (and not static nature).

P.164: In the first sentence, replace $\mathrm{d}\theta/\mathrm{d}t$ by $\mathrm{d}\theta/\mathrm{d}\tau$.

P.185: In the curvature tensor on the right-hand side of Equation 6.23, the order of the alpha and beta subscripts should be reversed to give $R^\mu{}_{\beta\alpha\gamma}$.

P.190: In Equation 6.27 the argument of the logarithm should have modulus signs rather than brackets i.e. $\ln\left|\dfrac{r}{R_\mathrm{S}} - 1\right|$.

P.195: In Figure 6.16 the rotation axis should pass through the points where the static limit and the outer event horizon meet.

Comment: The static limit is ellipsoidal, as shown, up to a certain limiting value of a and above that dips in at the poles (like an apple).

P.200: The value given for the Planck length at the bottom of the page should be $1.62 \times 10^{-35}\,\mathrm{m}$.

P.210: Replace Exercise 7.3 (b) with the following: 'Calculate the time dilation due to special relativity for a GPS satellite clock moving at a speed appropriate for its orbital radius, relative to a stationary observer.'

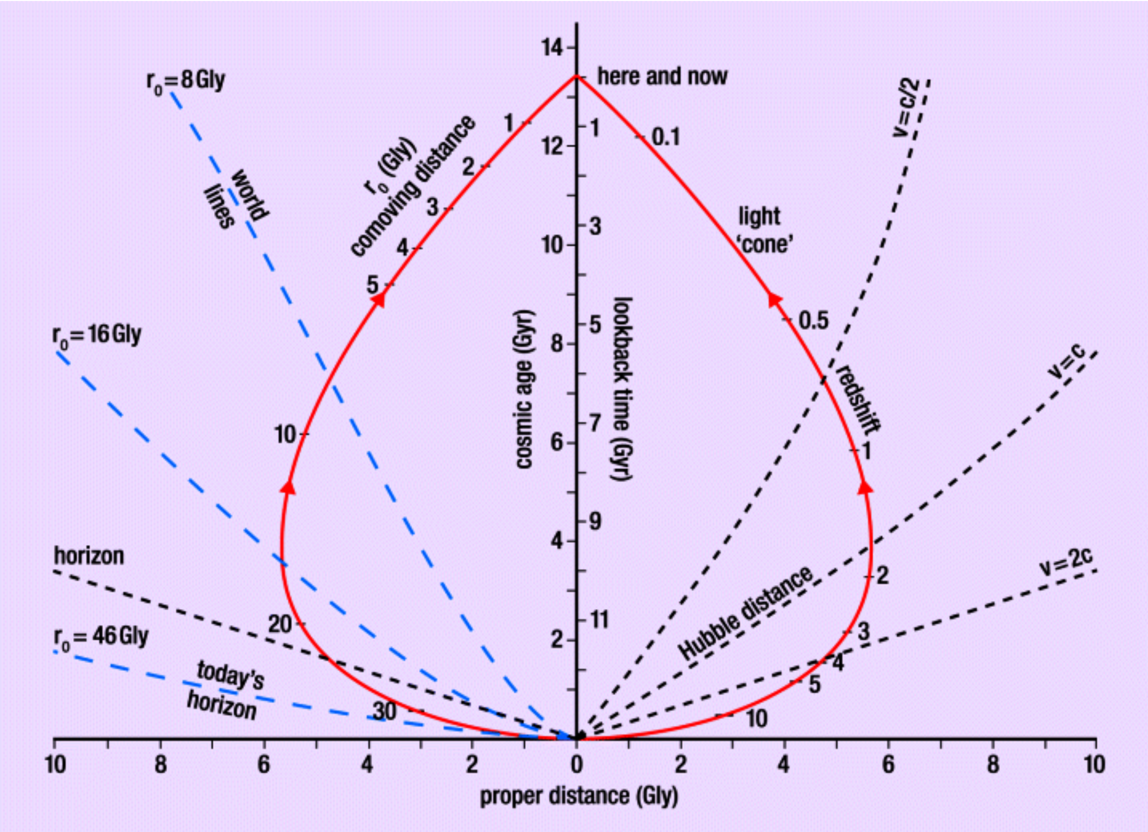
P.253: Equation 8.23 should read $p_\mathrm{m} = w\rho_\mathrm{m}c^2$.

P.262 and P.305: In Exercise 8.8 and its solution, the final expression should read $\Omega_{\Lambda,0} > \dfrac{\Omega_{\mathrm{m},0}}{2}$ (i.e. not $\geq$).

P.266 and P.267: This argument is correct only in the case of a flat Universe. In a universe with positive curvature, the radiation is spread over an area proportional to $(\sin(\chi))^2$, rather than χ^2. In a universe with negative curvature, this is instead $(\sinh(\chi))^2$. Equation 8.61 as given is the flat universe case, while for the more general case it will involve $\sin(\chi)$ or $\sinh(\chi)$. Nevertheless, Equation 8.61 will be correct to first order for small distances, so the remainder of the argument in the section will still hold.

P.268: In Figure 8.16, the horizontal axis should be labelled $d_\mathrm{L}\,/\,\mathrm{Mpc}$.

P.271: In Figure 8.17 the world-lines are very slightly incorrect. This figure was adapted from Mark Whittle, University of Virginia in Ellis, G (2008) 'Opposing the multiverse', *Astronomy and Geophysics*, Vol. 49, April 2008 and the original (correct) version can be seen below. This diagram should only be used for the world-lines and for all other purposes please use the diagram in the book, which is drawn to OU standards and therefore has the correct labelling and units:



P.280: In the solution to Exercise 1.5, the final quantity in the table should be $-L_P$ and not L_P (since $\Delta x'$ must be negative). Consequently, in the last line but one of the solution, $L_P = \ldots$ should be replaced by $-L_P = \ldots$. The last line of the solution is correct as it stands.

P.288: At the end of the solution to Exercise 3.9(b) it should read '... and Equation S3.2 becomes $0 + 2 \times 0 \times 0 \times 1 = 0$' This does not affect the final result.

P.300: In the solution to Exercise 7.1, using the values given leads to an answer of $43.00''$ per century (and not 42.99).

P.301/302: The solution to Exercise 7.3 has a couple of errors, but the final result is correct. In part (a), the comparison should be between the proper time intervals of the ground-based clock and the satellite clock, their coordinate time intervals are the same. As a result, the time difference comes out as +45.7 microseconds. For part (b), the moving clock measures a shorter interval, so the time difference comes out as −7.2 microseconds. The full solution should read as follows:

(a) Let $R_\oplus = 6371.0\,\text{km}$ be the mean radius of the Earth, $M_\oplus = 5.9736 \times 10^{24}\,\text{kg}$ be the mass of the Earth, and $h = 20200\,\text{km}$ be the height of the satellite above the Earth. From Equation 5.14, the proper time interval at $R_\oplus$ and the proper time interval at $R_\oplus + h$ are related by

$$\frac{\Delta\tau_{R_\oplus+h}}{\Delta\tau_{R_\oplus}} = \left(\frac{1-\frac{2M_\oplus G}{c^2(R_\oplus+h)}}{1-\frac{2M_\oplus G}{c^2 R_\oplus}}\right)^{1/2}$$

where the coordinate time at the two locations is the same and has cancelled.

Since the time dilation is small, we can use the first few terms of a Taylor expansion to evaluate this. Putting $2M_\oplus G/c^2(R_\oplus + h) = x$ and $2M_\oplus G/c^2 R_\oplus = y$, the right-hand side above becomes $(1-x)^{1/2} \times (1-y)^{-1/2}$. By a Taylor expansion, this is approximately $(1-\frac{x}{2})(1+\frac{y}{2}) \approx 1 - \frac{x}{2} + \frac{y}{2}$. So we have

$$\Delta\tau_{R_\oplus+h} \approx \left(1 - \frac{M_\oplus G}{c^2(R_\oplus + h)} + \frac{M_\oplus G}{c^2 R_\oplus}\right)\Delta\tau_{R_\oplus} = \Delta\tau_{R_\oplus} + \frac{M_\oplus G h}{c^2 R_\oplus (R_\oplus + h)}\Delta\tau_{R_\oplus}.$$

The discrepancy over 24 hours is given by

$$\begin{aligned}\Delta\tau_{R_\oplus+h} - \Delta\tau_{R_\oplus} &= +\frac{5.9736\times10^{24}\times6.673\times10^{-11}\times2.02\times10^{7}}{(2.998\times10^{8})^2\times6.371\times10^{6}\times(6.371+20.2)\times10^{6}}\times24\times3600\,\mathrm{s}\\ &= +45.7\mu\,\mathrm{s}.\end{aligned}$$

The positive sign indicates that the effect of general relativity is that the satellite clock runs more rapidly than a ground-based one (i.e. more time elapses according to the satellite clock).

(b) Special relativity relates an observed time interval Δt for a clock moving at a speed v with the time interval Δt_0 in the frame of reference in which the clock is at rest by

$$\Delta t = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}\Delta t_0.$$

For a satellite of mass m orbiting the Earth at a distance h from the Earth's surface, its speed v is given by

$$\frac{GM_\oplus m}{(R_\oplus + h)^2} = \frac{mv^2}{R_\oplus + h}$$

therefore

$$v^2 = \frac{GM_\oplus}{R_\oplus + h}$$

and hence

$$\Delta t = \left(1 - \frac{GM_\oplus}{c^2(R_\oplus + h)}\right)^{-1/2}\Delta t_0 \approx \left(1 + \frac{GM_\oplus}{2c^2(R_\oplus + h)}\right)\Delta t_0.$$

Hence the discrepancy over 24 hours between the satellite- and ground-based clocks is

$$\begin{aligned}\Delta t_0 - \Delta t \approx -\frac{GM_\oplus}{2c^2(R_\oplus + h)}\Delta t_0 &= -\frac{6.673\times10^{-11}\times5.9736\times10^{24}}{2\times(2.998\times10^{8})^2\times(6.371+20.2)\times10^{6}}\times24\times3600\,\mathrm{s}\\ &= -7.2\mu\,\mathrm{s}.\end{aligned}$$

The negative result indicates that the effect of special relativity is that the satellite clock runs slower than a ground-based one (i.e. less time elapses according to the satellite clock).

Comment: In reality, ground based clocks are moving too, at a speed due to the Earth's rotation, which varies with latitude. Also the direction of motion of the orbiting satellite relative to that of the ground based clock needs to be taken into account, in order to determine the correct velocity component to use. So the calculation of the appropriate special relativistic correction is more complex than implied here.

(c) The total effect of the results obtained in parts (a) and (b) is a discrepancy between the ground-based and satellite-based clocks of $(+45.7 - 7.2) = +38.5\mu\,\mathrm{s}$ per day. Since the basis of the GPS is the accurate timing of radio pulses, over 24 hours, this could lead to an error in distance of up to $c\,(\Delta t_0 - \Delta t) = 2.998\times10^{8}\times38.5\times10^{-6}\,\mathrm{m} = 11.5\,\mathrm{km}$.

Observational Cosmology

P.15: In the final paragraph of the solution to Worked Example 1.1, the total interval should be given as $0.5c$ seconds and not $0.5c$ metres.

P.21: In the final paragraph, replace the first sentence with 'Suppose that the distance between us and a distant galaxy at some time is $\ell = D \times R$, where R is the scale factor of the Universe at that time, and D is some constant.'

P.38: In item 4 of the Chapter 1 summary, 'consistent with special relativity' is superfluous and should be removed.

In item 5, 'proper motions' should be 'peculiar motions'.

P.49: In the first line of the third paragraph it should read '...decay with a mean lifetime of $885.7 \pm 0.8\,\mathrm{s}$.' (i.e. not half-life).

P.89: In summary point 7, the second sentence should read 'The observed abundances agree with predictions, ...'.

P.95: In the first margin note, the Sandage reference should be 2010 and not 2009.

P.105: In the first paragraph of Section 3.10 it should read '...from which one can calculate a luminosity distance or an angular diameter distance, respectively.'

P.108: In the margin note, the author reference should be Riess and not Reiss.

P.133: In the first sentence of the final paragraph, $z > 4$ should read $z > 3$.

P.136: In the paragraph below Equation 4.20 it should read '...a particular sort of galaxy is detected in the volume, ...'.

P.153: In the last sentence of the caption to Figure 4.27, it should say 'The panels at the bottom ...'.

P.159: In Figure 5.1, the value at the bottom of the y-axis should be 0.01.

P.161: In the line above Equation 5.4 the units for energy flux density should be $\mathrm{J\,s^{-1}\,Hz^{-1}\,sr^{-1}\,m^{-2}}$.

P.183 and P.252: In the footnote on p.183 and the second bullet point in the 'Further reading' section on p.252, the Lambourne reference should be to *Relativity, Gravitation and Cosmology* (and not *Theoretical Cosmology*).

P.186: In the line below Equation 6.3 it says that the Thomson scattering cross section is proportional to q^2/m^4 but this should be q^4/m^2.

P.187 (Applies to Edition 1.2 only. For Edition 1.1 see the other Errata pdf document.) The sentence immediately before the final paragraph should read: 'So the e-folding timescale for the growth of the black hole is $t_\mathrm{E}\dfrac{\eta}{1-\eta}$.

P.197: At the end of line 11 it should read '...followed by $(3.2 \pm 0.9) \times 10^9\,\mathrm{M_\odot}$ ...' (and not $\pm 10^9\,\mathrm{M_\odot}$).

P.204: In line 10 it should read '...two important *observational* consequences.'

P.223: In the 3-momentum volume on the right-hand side of Figure 7.8, the axis labelled p^3 should be labelled p^0. Also, the second line of the expression for V_p should read: $(p^2)^2\,\Delta p^2\,\Delta\Omega$.

P.273: The final sentence should read: 'He II is harder to ionize than hydrogen'.

P.307: Replace the solution to Exercise 7.1 with the following:

Comoving distances add, so $r_\mathrm{S} = r_\mathrm{L} + r_\mathrm{LS}$ Therefore $r_\mathrm{LS} = r_\mathrm{S} - r_\mathrm{L}$. Angular diameter distances are $D_\mathrm{S} = r_\mathrm{S}/(1+z_\mathrm{S})$ and $D_\mathrm{L} = r_\mathrm{L}/(1+z_\mathrm{L})$. The angular diameter distance of the source as seen from the lens is $D_\mathrm{LS} = r_\mathrm{LS}/(1+z_\mathrm{LS})(1+z_\mathrm{L})$, noting the extra $(1+z_\mathrm{L})$ factor because the Universe was smaller by this factor at that time, compared to now. Here z_LS is the redshift of the source seen by the lens.

Now $(1+z_\mathrm{S}) = (1+z_\mathrm{L}) \times (1+z_\mathrm{LS})$

Therefore the angular diameter distance $D_\mathrm{LS} = r_\mathrm{LS}/(1+z_\mathrm{S}) = (r_\mathrm{S} - r_\mathrm{L})/(1+z_\mathrm{S})$.

Extreme Environment Astrophysics

P.11: The final sentence before Equation 1.2 should read: '... and so $E_{\rm GR}(r = \infty) \ldots$.' i.e. the subscript GR is missing.

P.29: The caption to Figure 1.15 should read 'The dashed curve is the best-fitting elliptical orbit ...'.

P.31: In the unnumbered equation following Equation 1.21, it should read $\frac{d\dot{E}_{\rm GR}}{dr}$ (i.e. there should be a dot above $E_{\rm GR}$).

P.42: In the 4th line it should read '... spectral type and evolutionary state ...' (i.e. not 'end').

P.64: In the third line of the first paragraph it should say AM Canum Venaticorum.

P.81: In Figure 3.9 the label ω_* should be ω_1 as in the figure caption.

P.81: The sentence 3 lines above Equation 3.23 should read: 'Dividing both sides by $(GM)^{1/2}$ and solving for $\nu_{\rm vis}\Sigma$ gives ...' i.e. replace GM by $(GM)^{1/2}$.

P.113: The penultimate sentence of the penultimate paragraph should read: '... in between these values, $\dot{M}_B < \dot{M} < \dot{M}_D$, then a stable state doesn't exist!'

P.156: In equation 6.11 replace $=$ by $\propto$.

P.165: The first sentence should read 'For instance, in the high state ...'.

P.254: In the solution to Exercise 1.10, the units of the speed of light, c, should be $\rm m\,s^{-1}$ (and not m).

P.267: For clarity, the first sentence of the solution to Exercise 5.3 should be changed to read: 'For the compact binary disc, the inner disc radius is close to the compact star's radius. This is $r_{\rm in} \simeq 10^{-2}\,\rm R_\odot$ (i.e. 10^7 m) for a white dwarf and 10^4 m for a neutron star.'

P.271: In the solution to Exercise 6.4, the expression in square brackets in the final equation should read $[(-4)^2 + (0)^2]$.

P.277: In the solution to Exercise 8.3, the unit of the Thomson cross-section given in the numerator of the equation should be $\rm m^2$ and not $\rm m^{-2}$.